Control Flow Analysis

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https://synthesis.to

Why?

Motivation

- **high-level** structure of a function
- detect branches and **loops**
- **pattern matching** to spot interesting code parts
- foundation for **automated** program analysis

Basic Block

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- sequence of **ordered** instructions

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- **single entry**: only first instruction can be target of a branch
- **single exit**: only last instruction can branch to other basic blocks

Basic Block Identification

Rules: Leader Instruction Identification

1. first instruction is a leader
2. target of a control flow transfer is a leader
3. instruction that immediately follows a control flow transfer is a leader

Split on calls?

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Know how your tool handles it.

Basic Blocks

```
; leader: first instruction
0x170A0: cmp edi, 26h
0x170A3: jz  short 0x170C0

; leader: follows a control flow transfer
0x170A5: jg  short 0x170B8

; leader: follows a control flow transfer
0x170A7: xor eax, eax
0x170A9: cmp edi, 10h
0x170AC: jz  short 0x170C2

; leader: follows a control flow transfer
0x170AE: cmp edi, 16h
0x170B1: setnz al
0x170B4: retn

; leader: target of control flow transfer
0x170B8: cmp edi, 5Fh
0x170BB: setnz al
0x170BE: retn

; leader: target of control flow transfer
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```
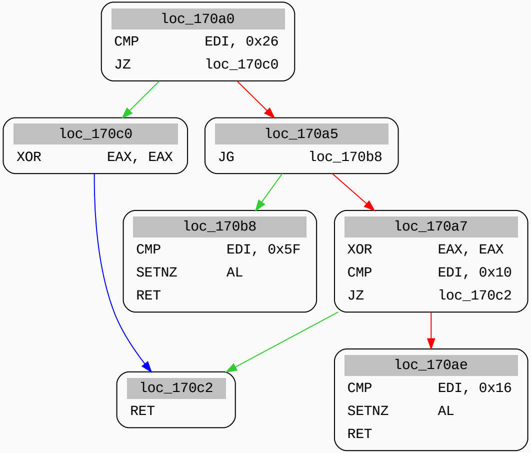

Control Flow Graph



Control Flow Graph

Control Flow Graph

- directed multigraph
- *nodes* are basic blocks
- *edges* represent control flow between basic blocks
- represents **all program paths** that might be traversed

Control Flow Graph

Entry
A node that has no incoming edges.

Control Flow Graph

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Exit

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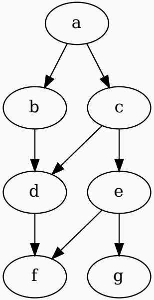
Path

A chain of transition between nodes.

Control Flow Graph



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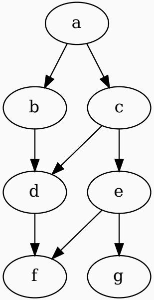

- a is a **entry** node
- f and g are **exists**
- $a \to c \to d \to f$ is a **path** between a and f

Dominance Relations

Motivation

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- loop detection and analysis
- foundation for many **compiler optimizations** and other analysis techniques

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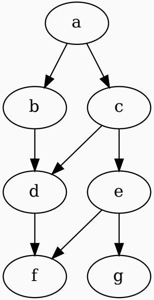
- **entry** node dominates **all** nodes in the graph

Dominator Sets



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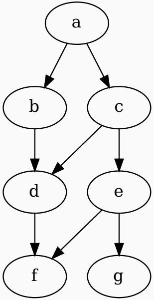

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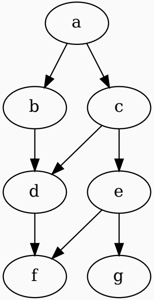

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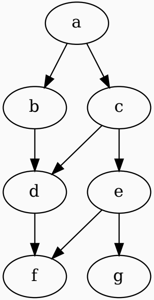

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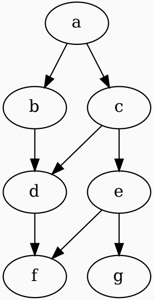

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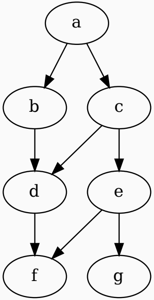

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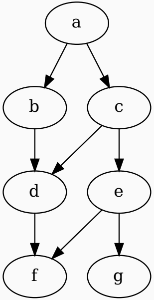

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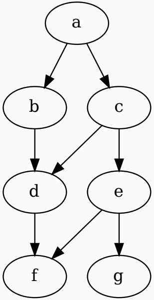

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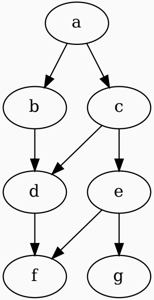
- **every** node (except entry) has an immediate dominator

Immediate Dominators



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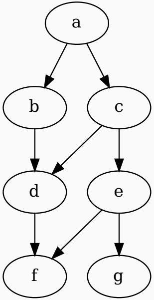

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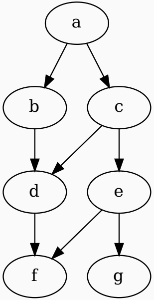

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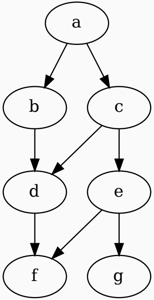

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- each node dominates its descendants in the tree

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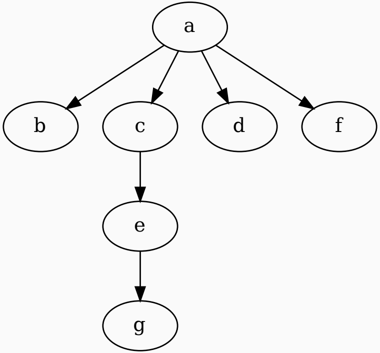

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What are loops and how can we find them?

Strongly Connected Component

A subgraph in which **each** node is **reachable** from **every** other node.

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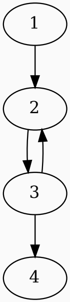
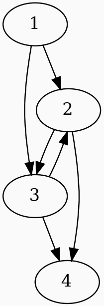
We focus only on natural loops.

Natural and Irreducible Loop



natural loop



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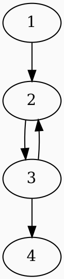
- **loop body**: set of **nodes within** a loop

Natural Loop



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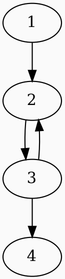

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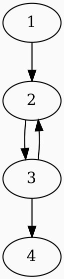

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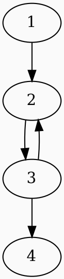

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- $(3, 2)$ is **back edge** to the dominator

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Natural Loop Detection

- find a back edge
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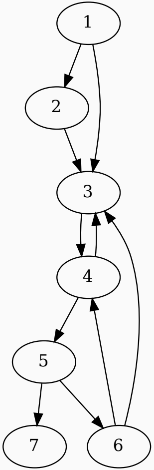
  2. there is an edge (y, x)
- identify the loop body
  1. collect all **nodes** that are **dominated by** x
  2. **filter** nodes that can **reach** y without visiting x

Natural Loop Detection



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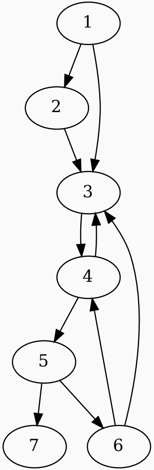
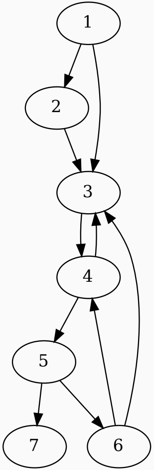


edge	body



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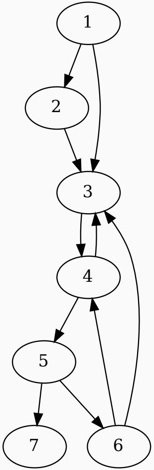

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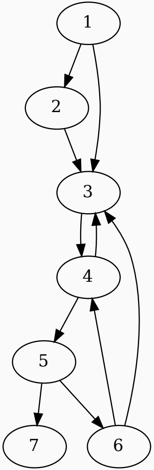

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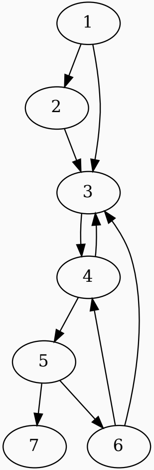

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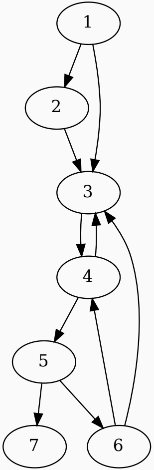

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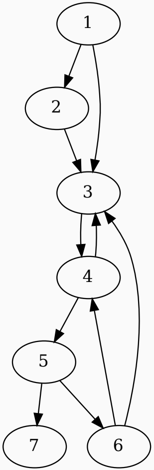

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Nesting Relations

Loops can be

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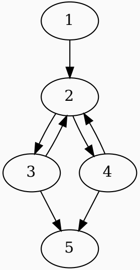
  - one function body is **entirely contained** within the other

Merged Loops



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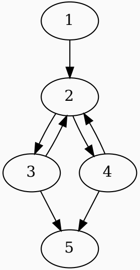

l_1: $\{2, 3\}$

l_2: $\{2, 4\}$

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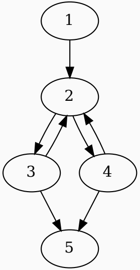
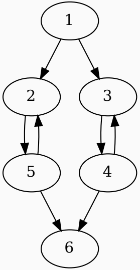


l_1: $\{2, 3\}$

l_2: $\{2, 4\}$

$l_1 \cap l_2 = \{2\}$

Disjoint Loops



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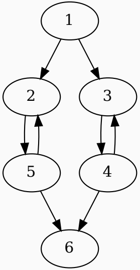

l_1: $\{2, 5\}$

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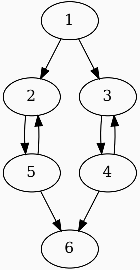
Disjoint Loops



l_1: $\{2, 5\}$

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$l_1 \cap l_2 = \emptyset$

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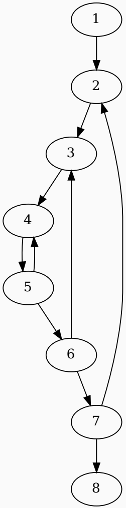

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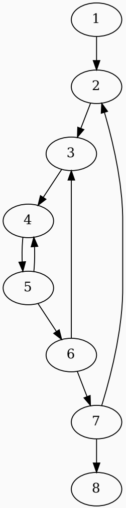
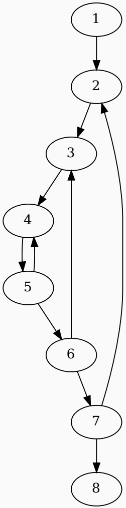


l_1: $\{4, 5\}$

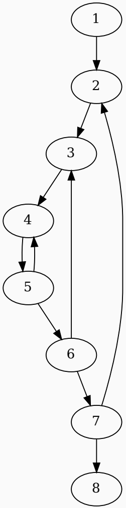
Nested Loops



l_1: $\{4, 5\}$

l_2: $\{3, 4, 5, 6\}$

Nested Loops

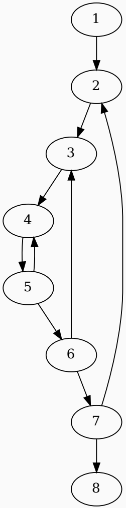


l_1: $\{4, 5\}$

l_2: $\{3, 4, 5, 6\}$

l_3: $\{2, 3, 4, 5, 6, 7\}$

Nested Loops



l_1: $\{4, 5\}$ innermost loop

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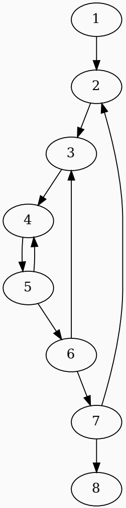

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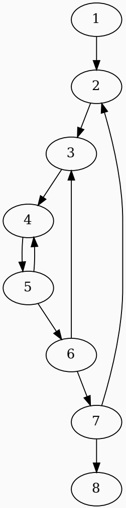
Nested Loops



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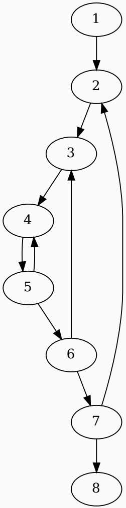
l_3:	$\{2, 3, 4, 5, 6, 7\}$	outermost loop

Nested Loops



l_1: $\{4, 5\}$ innermost loop

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l_3: $\{2, 3, 4, 5, 6, 7\}$ outermost loop

$l_1 \subset l_2 \subset l_3$

Loop Unrolling

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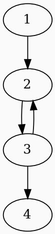
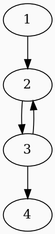
- transformed graph is **semantically equivalent** for up to k loop iterations

Loop Unrolling



natural loop

Loop Unrolling



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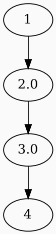
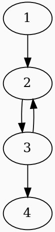


unrolling depth 0

Loop Unrolling



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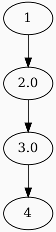

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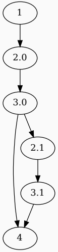

unrolling depth 1

Conclusion

Control Flow Analysis

- basic blocks
- control flow graph construction
- dominance relations
- natural loop detection
- loop properties and transformations